Simultaneous Estimation of Sparse Signals and Systems at Sub-Nyquist Rates

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EXTENDED ABSTRACT

In this work, we consider the problem of system identification based on a sparse sampling system. Unlike standard techniques for system identification which require the sampling rate to be at or above the Nyquist rate, we use sparse sampling techniques to identify the system at sub-Nyquist sampling rates. We propose a novel algorithm for simultaneous estimation of sparse signals along with system identification using the theories of finite rate of innovation (FRI) sampling [3], [1]. Specifically, we will divide the estimation problem into two stages where we first assume that the input sparse signal is known, so that the problem simplifies to a system identification problem only and then in the second stage, we consider the problem of simultaneously estimating the input sparse signal and also the linear system, known as blind system identification, and propose a novel iterative algorithm for that setup. We will show that, based on our numerical simulations, the solution to the second problem is normally convergent.

System Identification with Known Input Signal

For this scenario, as shown on Figure 1, a two-channel system is proposed for sampling the input sparse signal with and without the unknown system. In the figure, $g(x)$ represents the known input signal, $\psi(x)$ represents the unknown system to be identified, $\varphi(x)$ represents the pre-defined sampling kernel which we assume to be purely imaginary E-splines [2] in both channels, T represents the sampling interval and s_k represent the samples. In the first channel, the input signal is

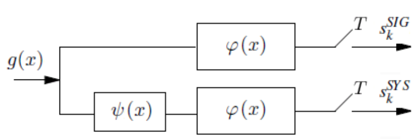

Fig. 1. System identification setup with known input signal

directly sampled with the kernel $\varphi(x)$ and given the obtained samples which we denote with s_k^{SIG}, the exponential moments of the input signal, denoted with τ_m^{SIG}, are calculated [1]. In the second channel, the same input signal is fed through the unknown system $\psi(x)$ and then sampled with the same sampling kernel. Same as in the first channel, given the samples s_k^{SYS}, the exponential moments τ_m^{SYS} are calculated.

With purely imaginary E-spline sampling kernel employed, by dividing the exponential moments obtained from the two channels, it can be shown that the Fourier transform of the unknown function can be obtained. Given the partial Fourier transform of the unknown system, there will be an inverse problem to solve for the unknown parameters of the unknown system. In our work, we show for cases such as finite impulse response (FIR) filters (e.g. acoustic room impulse response estimation or line echo cancelation), B-splines (e.g. camera lens calibration) and E-splines (e.g. estimation of the electronic components of a finite order electronic circuit), we can solve the above inverse problem and identify the system. It should be pointed out that the above method works regardless of the structure of the input signal.

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Blind System Identification

When both the signal and the system are unknown, the previous solution cannot be used directly and the problem is in general more involved. However, a recursive version of the discussed method can be utilized to estimate both the input sparse signal and the unknown system.

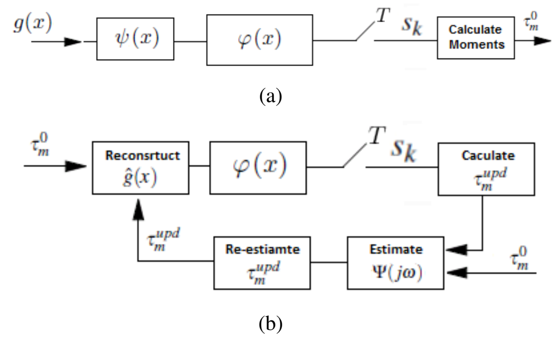

Fig. 2. The setup proposed for recursive estimation

In our work we assume that the input sparse signal is a stream of Diracs with unknown locations and amplitudes. As shown in Figure 2(a), the unknown input signal is fed to the unknown system $\psi(x)$ and then is sampled with our pre-specified purely imaginary E-spline sampling kernel. The annihilating filter method [3], [1] is directly applied to the exponential moments τ_m^0 and an initial estimate of the input signal is obtained, denoted as $\hat{g}(x)$ (Figure 2(b)). The estimated signal $\hat{g}(x)$ is recursively fed back to sampling kernel and its corresponding updated exponential moments are calculated at each recursion, denoted with τ_m^{upd}. By dividing the updated exponential moments τ_m^{upd} and the initial measurements τ_m^0, an estimate of the Fourier transform of the unknown system is obtained. From this estimate, the unknown parameters of the unknown system are estimated and the measurements τ_m^{upd} are re-calculated. Our empirical results show that by applying the above method recursively, the estimations converge to the actual input signal $g(x)$ and the unknown function $\psi(x)$.

REFERENCES

[1] P.L. Dragotti, M. Vetterli, and T. Blu. Sampling moments and reconstructing signals of finite rate of innovation: Shannon meets Strang-Fix. *In IEEE Transactions on Signal Processing*, 55(5):1741–1757, May 2007.

[2] M. Unser and T. Blu. Cardinal Exponential Splines: Part I-theory and filtering algorithms. *In IEEE Transactions on Signal Processing*, 53(4):1425–1438, April 2005.

[3] M. Vetterli, P. Marziliano, and T. Blu. Sampling signals with finite rate of innovation. *In IEEE Transactions on Signal Processing*, 50(6):1417–1428, June 2002.